

# Small Signal Analysis of Active Circuits Using FDTD Algorithm

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**Abstract**—The FDTD method is extended to analyze a microwave amplifier. This amplifier includes matching circuits, DC bias circuits, and an active device. Equivalent current sources are used to model the active element. With the small signal model of the active element, the FDTD full-wave simulations show good agreement with measured results.

## I. INTRODUCTION

THE Finite-Difference Time-Domain (FDTD) method is a powerful and versatile numerical method for the simulation of Maxwell's equations. It has been widely used to analyze linear and passive structures. Many practical microwave circuits, however, are embedded with components which are both active and nonlinear (e.g., semiconductor devices), and future electromagnetic field simulations will require the modeling of these circuits. Time-domain analysis is well-suited for analyzing such system characteristics. Recently, the FDTD method has been extended to microwave active circuits. In [1], the FDTD method was extended to incorporate an equivalent active region of a Gunn diode. Instead of using an active region, a lumped equivalent circuit was used to model the active device in [2], [3], which allows direct access to the SPICE circuit simulator.

In this paper, the FDTD method is applied to analyze a microwave amplifier circuit. The amplifier is typical and contains matching circuits, DC biasing circuits, and a three-terminal active device. The active device is characterized by its small signal equivalent circuit model and treated as a lumped element in the simulation.

## II. MODEL

The system under consideration is shown in Fig. 1. The entire system contains three types of structures: distributed passive structures, lumped passive devices, and an active device (GaAs MESFET). It is apparent that the distributed passive structures can be simulated using the FDTD algorithm by dividing the structures into cells [4]. Moreover, the lumped

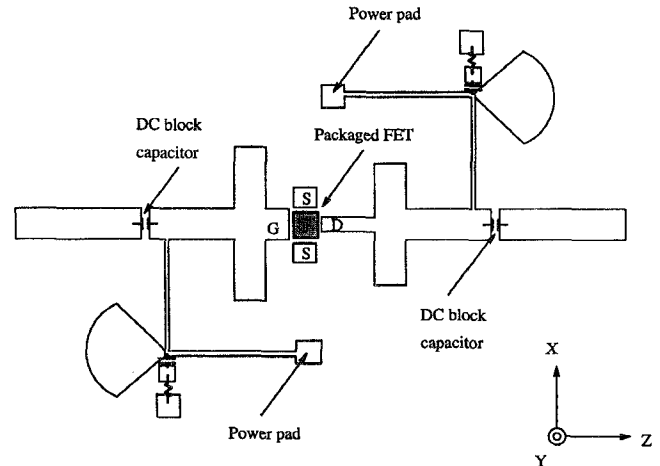


Fig. 1. The layout of the microwave amplifier, which is designed at 6 GHz with 9 dB gain.

passive devices also can be treated as distributed elements and absorbed into coefficients of the FDTD algorithm [5].

To consider the two-port active device in FDTD simulation, the device is substituted with equivalent current sources. These sources represent the current-voltage relationship at the device terminal and characterize the input impedances as well as the transfer functions of the active device. The interaction between the equivalent current sources and the electromagnetic field is governed by Ampere's current law [2]

$$C \frac{dV}{dt} + I_{device}(V) = I_{total} \quad (1)$$

where  $C$  is the equivalent capacitance of the FDTD cell,  $I_{device}$  represents the equivalent current sources,  $V$  is the terminal voltage of the active device, and  $I_{total}$  means the total current from the integration of  $H$  fields.

The equivalent current sources are determined by the small signal lumped circuit model of the active device. Here, the circuit theory is applied to solve the state equation of the equivalent circuit of (1). Generally, the state equation is expressed in matrix form [6]

$$\mathbf{A} \cdot \frac{d\mathbf{X}(t)}{dt} = \mathbf{B} \cdot \mathbf{X}(t) + \mathbf{F}(t) \quad (2)$$

where the vector  $\mathbf{X}$  denotes the state variables, the elements of matrices  $\mathbf{A}$  and  $\mathbf{B}$  are determined by the circuit element values, and the forcing term  $\mathbf{F}$  comes from the total current. The state equation can be solved by the backward finite

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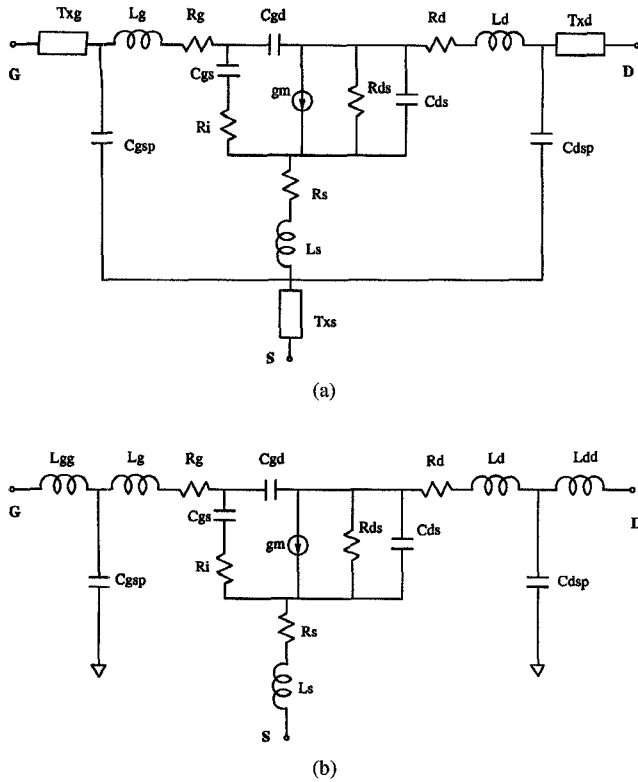


Fig. 2. Lumped equivalent circuits of the GaAs MESFET. (a) The packaging effect is modeled with ideal transmission lines, and (b) the packaging effect is modeled with inductors and capacitors.

difference scheme in each time advance, or with the circuit simulator SPICE. Using the same time step as that in the FDTD algorithm, terminal voltages of the active device are fed back into the FDTD simulation.

### III. RESULTS

The amplifier is designed to have 9 dB gain at 6 GHz. Two of the lumped circuit models used in this paper for the GaAs MESFET, NEC72084, are depicted in Fig. 2. The difference in element arrangements is the modeling of the packaging effect. In Fig. 2(a), the effect is modeled with ideal transmission lines. A coupled FDTD-SPICE calculation is used for this circuit. In Fig. 2(b), the effect is modeled with inductors and capacitors. A differential equation solver has been developed to calculate the state variables. The element values of both models are optimized to match the measured S-parameter of the active device biased at  $V_D = 3$  V and  $I_{DS} = 30$  mA.

FDTD simulations are performed with uniform grids. The third-order absorbing boundary conditions are used as in [7]. Power pads connect to the ground plane through vias to model RF short circuits. In the calculation using the differential equation solver, the space steps used are  $\Delta x = 13$  mil,  $\Delta y = 15$  mil, and  $\Delta z = 10$  mil. The total mesh dimensions are 91 (width)  $\times$  12 (height)  $\times$  195 (length) in  $x$ ,  $y$ , and  $z$  directions, respectively. The packaged MESFET occupies 8 FDTD cells in the longitudinal direction and has contacts which extend over the entire width of the microstrip lines. The equivalent current sources are placed at those cells connected to the microstrip line. The voltages from the gate to the

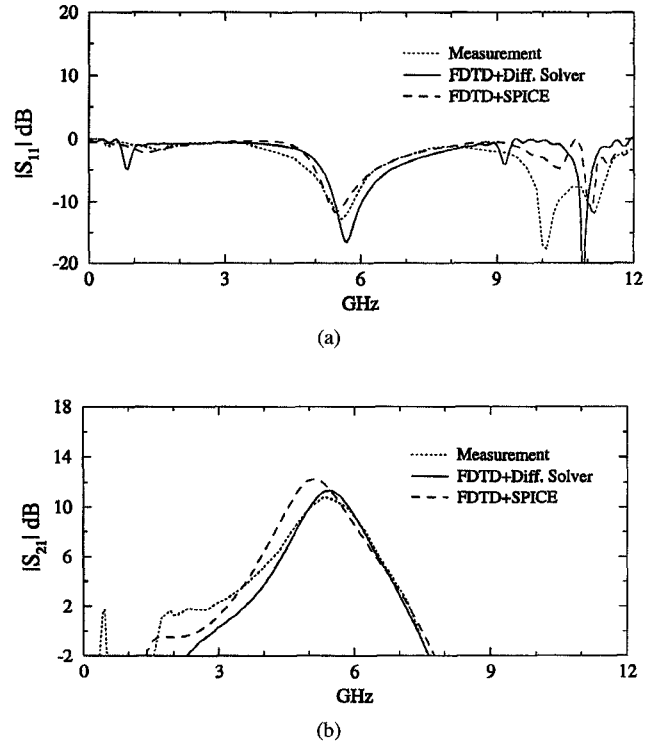


Fig. 3. The S-parameter of measured data and the calculated results from FDTD simulations using equivalent circuits in Fig. 2. The gain at 6 GHz and the match point for both cases, using coupled FDTD-SPICE with the model in Fig. 2(a) and a differential solver with the model in Fig. 2(b), agree well with measured data.

ground plane and from the drain to the ground plane are used, while the source voltage is not explicitly used in FDTD calculations. For calculating the two-port S-parameter, the reference planes are  $30 \Delta z$  from the DC-block capacitors. In the coupled FDTD-SPICE calculation, the ideal transmission lines in Fig. 2(a) are implemented using LC ladders with the same electrical characteristics as the transmission lines. For this application, two elements of LC ladders are sufficient. The space steps chosen are  $\Delta x = 5.2$  mil,  $\Delta y = 5$  mil, and  $\Delta z = 5$  mil, and the mesh dimensions are  $267 \times 15 \times 311$ . There are 16 longitudinal cells for the active element. Fig. 3 displays measured data and the calculated S-parameter by taking Fourier transform of the observed time responses in the simulations.

The gain at 6 GHz, using the differential solver, is 9.3 dB and, using the FDTD-SPICE calculation, is 8.5 dB. The measured gain is 9.23 dB. The match point is 5.7 GHz with the differential solver, 5.43 GHz with the FDTD-SPICE calculation, and 5.6 GHz with measurement. Because of different models of the MESFET device, there is some discrepancy between both results. The simulations, however, agree well with measured results at the designed frequency for both cases. Some out-of-band dips in measurement near 1 and 11 GHz due to DC biasing circuits also appear in the simulation, as shown in Fig. 3.

### IV. CONCLUSION

An equivalent current source approach has been developed for the time domain analysis of a practical microwave

amplifier. The calculated S-parameter is in good agreement with measured data. This approach is capable of modeling electromagnetic fields in a wide variety of microwave and millimeter-wave circuits. In addition, the time domain simulation also provides a clear viewpoint of the transient behavior and wave phenomena.

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